January 10, 1861.

Major-General SABINE, R.A., Treasurer and Vice-President, in the Chair.

The Right Hon. Sir William Erle was admitted into the Society.

The following communications were read:—

I. "On the Equation for the Product of the Differences of all but one of the Roots of a given Equation." By ARTHUR CAYLEY, Esq., F.R.S. Received November 30, 1860.

(Abstract.)

It is easy to see that for an equation of the order n, the product of the differences of all but one of the roots will be determined by an equation of the order n, the coefficients of which are alternately rational functions of the coefficients of the original equation, and rational functions multiplied by the square root of the discriminant. In fact, if the equation be $\phi v = (a, \ldots \between v, 1)^n = a(v-\alpha)(v-\beta)\ldots$, then putting for the moment $a=1$, and disregarding numerical factors, $\sqrt{\square}$, the square root of the discriminant, is equal to the product of the differences of the roots, and $\phi'\alpha$ is equal to $(\alpha-\beta)(\alpha-\gamma)\ldots$, consequently the product of the differences of the roots, all but α, is equal to $\sqrt{\square} \div \phi'\alpha$, and the expression $\frac{1}{\phi'\alpha}$ is the root of an equation of the order n, the coefficients of which are rational functions of the coefficients of the original equation. I propose in the present memoir to determine the equation in question for equations of the orders three, four, and five: the process employed is similar to that in my memoir "On the Equation of Differences for an equation of any Order, and in particular for Equations of the Orders Two, Three, Four, and Five," Phil. Trans. t. cl. p. 112 (1860), viz. the last coefficient of thegiven equation is put equal to zero, so that the given equation breaks up into $v=0$ and into an equation of the order $\overline{n-1}$ called the reduced equation; and this being so, the required equation breaks up into an equation of the order $\overline{n-1}$ (which, however, is not, as for the equation of differences, that which corresponds to the reduced equation) and into a linear equation; the

equation of the order $\overline{n-1}$ is calculated by the method of symmetric functions; and combining it with the linear equation, which is known, we have the required equation, except as regards the terms involving the last coefficient, which terms are found by the consideration that the coefficients of the required equation are seminvariants. The solution leads immediately to that of a more general question; for if the product of the differences of all the roots except a, of the given equation $\phi v = (*\between v, 1)^n = a(v-a)(v-\beta)\ldots = 0$ (which product is a function of the degree $n-2$ in regard to each of the roots $\beta\,\gamma\delta\ \ .$), is multiplied by $(x-ay)^{n-2}$, the function so obtained will be the root of an equation of the order n, the coefficients of which are covariants of the quantic $(*\between x,y)^n$, and these coefficients can be at once obtained by writing, in the place of the seminvariants of the former result, the covariants to which they respectively belong. In the case of the quintic equation, one of these covariants is, in regard to the coefficients, of the degree 6, which exceeds the limit of the tabulated covariants, the covariant in question has therefore to be now first calculated. The covariant equations for the cubic and the quartic might be deduced from the formulæ Nos. 119 and 142 of my Fifth memoir on Quantics, Phil. Trans. t. cxlviii. pp. 415–427 (1858); they are in fact the bases of the methods which are there given for the solution of the cubic and the quartic equations respectively; and it was in this way that I was led to consider the problem which is here treated of.

II. "Description of a new Optical Instrument called the 'Stereotrope.'" By WILLIAM THOMAS SHAW, Esq. Communicated by WARREN DE LA RUE, Esq. Received Dec. 6, 1860.

This instrument is an application of the principle of the stereoscope to that class of instruments variously termed thaumatropes, phantascopes, phenakistoscopes, &c., which depend for their results on "persistence of vision." In these instruments, as is well known, an object represented on a revolving disc, in the successive positions it assumes in performing a given evolution, is seen to execute the movement so delineated; in the stereotrope the effect of solidity is superadded, so that the object is perceived as if in motion and with